

Modelling the folding motions of the curved folds

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Abstract

We propose a user interface to model and visualize curved folding with one curved crease on a sheet of paper. In the proposed system, the folded paper is represented as two quad strips adjacent via fold curve, and transition of rulings during fold or twist motion is supported as shown in Fig. 1. In recent studies on paper modelling and simulation, 3D surfaces generated by curved folding are often represented by quad strips with fixed geometry, and the folding motion is simulated by the method of rigid folding [1][2]. However, such methods do not support deformation where the directions of rulings change while the paper is folding. There is also a method to simulate a sheet of thin materials by adaptive mesh refinement [3]. This method may well model irregular deformations of paper such as crumpled paper or ruling transition by generating complicated mesh data composed of thousands of faces. To simulate the motion of the curved fold using simple model, our method places a quad strip on each side of the crease, and each quad in the strips having four edges: a crease, two rulings, and the boundary of the paper. As the rulings drift, the geometry of the quads are updated while the topology is consistent. The geometric relationship between the curves, folding angles, and the ruling directions are investigated by Fuchs and Tabachnikov [4] and introduced by Tachi [5] as follows:

$$k_{2D}(s) = k(s) \cos \alpha(s), \quad \cot \beta_L(s) = \frac{\alpha(s)' - \tau(s)}{k(s) \sin \alpha(s)}, \quad \cot \beta_R(s) = \frac{-\alpha(s)' - \tau(s)}{k(s) \sin \alpha(s)}, \quad (1)$$

where $\beta_L(s), \beta_R(s)$ are the angles between the curved crease and the rulings on left and right side, $\alpha(s)$ is the folding angle, $k(s)$ the curvature, $\tau(s)$ the torsion of the 3D curve, and $k_{2D}(s)$ the curvature of the 2D curved crease. Given a set of curved crease and 3D space curve, or a set of curved crease and the folding angle $\alpha(s)$, the ruling directions and the shape of the paper are calculated and the quads are updated. Our method has features to avoid crossing of the rulings and to keep the surface developable by automatic parameter adjustment of torsion and folding angle, as well as trimming of the paper defined by the user.

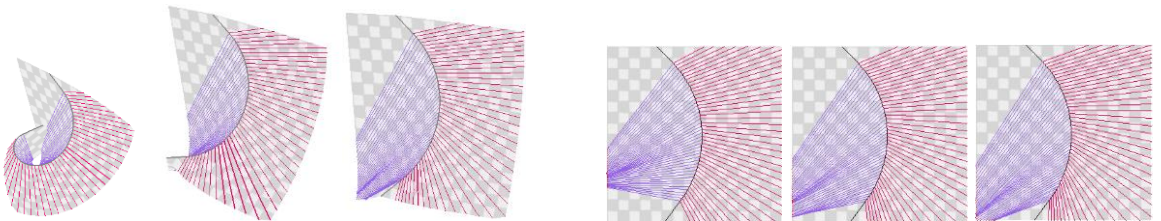


Figure 1: Transition of rulings as the paper being fold. (left: 3D shape, right: crease pattern)

In our system, the curved crease is initially defined by the user and then its final folded state is controlled by curvature, torsion and/or folding angle on some sampled points on the crease curve. The shape of the paper is calculated based on equation (1) by interpolating the values on rest of the points (Fig. 2(a)(b)). The users often need to adjust the parameters and re-calculate the shape so that the rulings do not cross each other (Fig. 2(c)). The area near the crease with very small curvature is regarded as one continuous plane and therefore rulings are omitted. Based on the nature of developable surface, torsion $\tau(s)$ and the difference of folding angle $\alpha'(s)$ are set to zero by the system where curvature is close to zero. To simulate the folding and unfolding motion of the curved folds, as in Fig. 2(d), folding angle and torsion is changed continuously between zero and the original value, which is the flat state and the final state of the paper, while other variables are re-calculated. In case of rulings crossing each other, user can trim the paper (Fig. 2(e)). The model is evaluated by visual comparison with a real paper, and checked its developability numerically by calculating the error between 2D crease pattern and 3D model.

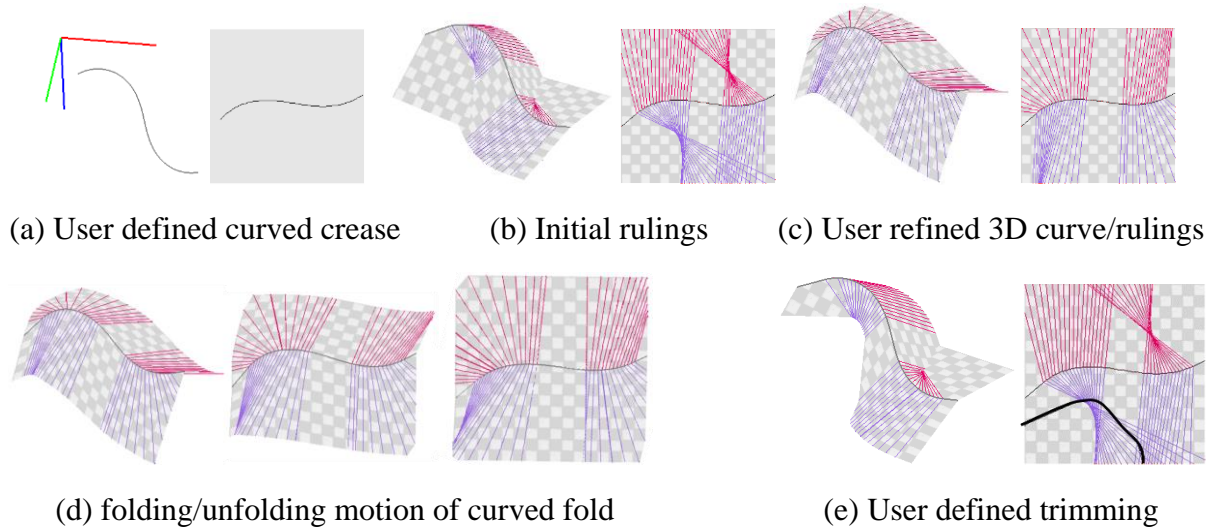


Figure 2: The curved fold designed by the user and simulated by our method.

- [1] M. Kilian, S. Flöry, Z. Chen, N. Mitra, A. Sheffer, and H. Pottmann. "Curved Folding". ACM Transactions on Graphics, 27(3):75:1-9, 2008.
- [2] T. Tachi. "Simulation of rigid origami". In Origami4: The Fourth International Conference on Origami in Science, Mathematics, and Education, pp.175–187, 2009.
- [3] R. Narain, T. Pfaff, and J. F. O'Brien. "Folding and Crumpling Adaptive Sheets". ACM Transactions on Graphics, 32(4):51:1–8, July 2013.
- [4] D. Fuchs, S. Tabachnikov, "More on Paperfolding". The American Mathematical Monthly, Vol. 106, No.1, pp. 27-35, 1999.
- [5] T. Tachi, "One-DOF Rigid Foldable Structures from Space Curves", in Proceedings of the IABSE-IASS Symposium 2011, London, UK, September 20-23, 2011.